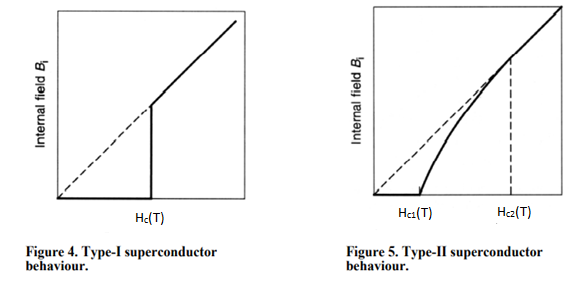
**Meisner Effect (Type I)**

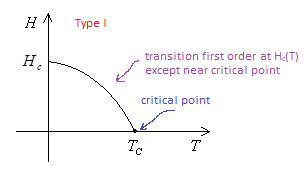
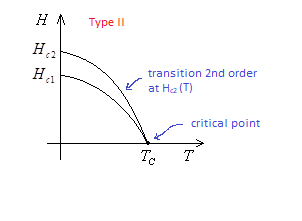
So a superconductor screens out magnetic fields. But as the field is ramped up, it will eventually break the degeneracy between the opposite spin Cooper pairs, making them energetically less favorable to form. And so the superconducting properties of the metal (namely Meisner effect – expulsion of all magnetic field lines, 0 resistance) will eventually vanish. This should start at Tc for even small fields probably. But even at T = 0 for large enough fields. Turns out how it eventually vanishes depends on the type of the superconductor. For Type I’s, at a given T < Tc, raising H = Bf/4π (‘faux’ Gaussian units, for suitable geometry, etc.) up from 0 to Hc(T) doesn’t change the number of Cooper Pairs in the superconductor. But once we cross Hc(T), then all Cooper Pairs dissassociate, the supercurrent dies, and flux is let in. And so apropos Type I’s, our GL Free energy only applies close to Tc, because it’s only there where |ψ|2 is small.

For Type II’s, the number of Cooper Pairs is constant from H = 0 to some critical Hc1(T), and then attenuates until vanishing completely at Hc2(T). Looking at the susceptibility curve below, we can see that even though we have a supercurrent present (because we still have Cooper pairs as asserted) between Hc1(T) and Hc2(T), flux is still allowed to penetrate the material, although less than normally would. Thus, it appears the Meisner effect isn’t realized. The resolution to this paradox seems to be that for type II’s, the Cooper pairs are confined to specific regions of the superconductor after Hc1 is passed so that even though they might carry a super current and can screen out the field within that region, they can’t screen it out outside that region. And this agrees with our conclusion in previous file that if ns were position-dependent, then the Meisner effect wouldn’t happen. The regions where the external field penetrates the sample are called vortices. And their radii goes from 0 at Hc1 to ∞ at Hc2. Hc2 is typically much larger than Hc1. So the transition from superconductor to normal conductor is 2nd order at Hc2. Maybe that’s why they’re called type II superconductors. Well this makes the entire Hc2 border amenable to analysis via the GL free energy since ψ is small.

The susceptibility diagram is:



The basic phase diagrams for type I, II superconductors are:

Just to be clear, we’re saying the order parameter |ψ|2 is small only near Tc for Type I’s, but is small along the entire Hc2(T) curve for type II’s.

**Meisner effect GL Free Energy**

So let’s just verify that our GL free energy does predict the Meisner effect. So in a previous file we introduced the GL (superconducting part of the) free energy,



where ψeq(x) is given by:



And recall we derived from this an equation of state,



(b = bound and is same as induced) So if we knew what ψeq were in the presence of A, we could calculate the bound current, and then the magnetic field as well. For now let’s simply presume some ψ with constant amplitude, but turns out we needn’t make any assumptions on the phase to get the Meisner effect, so we’ll let it be arbitrary: ψeq = √n\*s·eiφ(x). Let’s work out the current,



So this matches what we got in the London theory (though we didn’t have extra φ term in BCS theory – is this because of our gauge representation of **E** field through **A** alone?).



and the Meisner effect would be a consequence, presuming ns\* were non-zero. So it might be nice to get T-dependence of ns\*. So have to go back to the ψeq equation.



We can set A = 0 as we’re presuming magnetic field is expelled from our superconductor. And our presumption is also that ns\* is position-independent (if not, then B wouldn’t be expelled anyway), so we can get rid of ∇2. Then we have:



So when r < 0 we get spontaneous creation of Cooper pairs/super current. For what it’s worth, we’ll note that the phase of ψ(x) is completely unspecified. But I guess I’ll take it to be zero. Then from above, the current corresponding to this would be:



and so can say,



and of course,



and of course from our earlier work, the penetration depth would be (in faux-Gaussian units):



**Hc(T) near critical point for Type I’s**

Since Type I superconductors experience a first order phase transition at Hc(T), the entire border is not amenable to our GL free energy, which presumes the order parameter to be small. But it should apply along Hc(T) for T close to Tc, where |ψ|2 would still be small. Near here, as we increase H past Hc(T), the superconductor transitions to a normal conductor, which would mean that the free energy (action) becomes lower with ψ = 0 (and the field consequently penetrating the conductor because no supercurrent and so no screening), than it is with ψ non-zero (i.e. the value which minimizes the free energy and the field consequently substantially eliminated from the conductor’s interior because of screening – see Meisner effect). So we can figure out Hc(T) by comparing Free energies (actions). Our free energy (really action as of yet, since ψ(x) isn’t yet determined) is given by:



and,



But in our scenario, we’re holding not the total field constant, but the free external field constant. So it is the free energy with T and Bf = 4πH (assuming solenoidal geometry, homogeneous substance, and faux Gaussian units) as proper variables, that should be minimized. This free energy, which I’ll call F\* is related to F by a Legendre transformation. Recall from somewhere in the Free Energy file that we said,



and so,



Therefore, we can switch to the proper variable **H** by constructing:



(I abandoned the t argument just cause it’s kind of implicit) and then,



So can see F\* has the proper thermodynamic variables for our scenario (T, H). So as we said, the cross-over occurs when the F\* action is minimized by assuming a normal metal state, in which case ψ = 0 and the external field H penetrates the metal (basically without reduction, as we saw in the free metal file), rather than a superconducting state, in which case ψ(x) assumes the homogeneous value ψ0 which minimizes F\*, and B (as well as A) – see Meisner effect file – is screened out. In symbols,



Let’s work on the LHS,



where in the second line we use the fact that the metal is normal and so has negligible response to the external field, and so the total field is just the external field. And in the the third line just presume homogeneity so H should be constant. Now let’s do the RHS,



Now we can find the ψ0 by minimizing the RHS (of course we already did that above in previous section),



Plugging this back into the free energy we have:



So altogether then we’ll transition to a normal metal when,



Therefore, at least near the critical point, our phase separation line is given by:



Will observe that there is linear dependence near the critical point.